

PGS-251021
Autonomous Examination DEC-2025
SEMESTER- I
M.Sc. - MATHEMATICS
Paper – I (Advanced Abstract Algebra-I)

Time : 3 Hours

M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

Objective type questions Attempt any 10 questions out of 12 given below.

(10 x 01 = 10)

- (i) Which of the following abelian group has a composition services
 (a) Group of integers (b) Group of rationals
 (c) Group of integers module n (d) Group of real numbers
- (ii) If $|G|$ denotes order of group G . Then which of the following implies that G is nilpotent ?
 (a) $|G| = 6$ (b) $|G| = 9$
 (c) $|G| = 24$ (d) $|G| = 24$
- (iii) If M is an R -module and A, B are R -submodules of M , then.
 (a) $A \cap B$ is an R -submodule of M .
 (b) $A+B = \{a+b \mid a \in A, b \in B\}$ is an R -sumodule of M .
 (c) AXB is an R -submodule of M .
 (d) All of the above
- (iv) $\lambda \in F$ is called a characteristic root (or eigen value) of $T \in A(V)$ if -
 (a) T is singular (b) λI is singular
 (c) $\lambda I - T$ is non-singular (d) $\lambda I - T$ is singular
- (v) The number of non-zero diagonal entries in the smith normal form equal the ;
 (a) Determinant (b) Rank of the matrix
 (c) Both (a) and (b) (d) None of the above

(02)

- (vi) A sequence $(G_0, G_1, G_2, \dots, G_n)$ of subgroups of a group G is called a subnormal series of G if.....
- (vii) If two groups H and K are solvable then $H \times K$ is also.....
- (viii) An R -module M is called free if it has a
- (ix) A linear transformation $T \in A_F(V)$ is called nilpotent if.....
- (x) The index of nilpotency of the matrix

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ -2 & 1 & 1 \end{bmatrix}$$
 is
- (xi) If $T \in A(V)$ is nilpotent, then the invariants of T are unique.
 (True/False)
- (xii) The smith normal form can be used to classify finitely generated modules over a principal ideal domain (PID). [True/False]

Section-B

Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below.

(3 x 3 = 09)

- (i) Show that a group of order P^n (p -prime) is nilpotent.
- (ii) Let G be a group and let G' be the commutator subgroup of G . Then show that G' is a normal subgroup of G .
- (iii) Define R -module over ring R with an example.
- (iv) Obtain the smith normal form and rank for the matrix with integral entries.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$$
- (v) Show that the abelian group generated by x_1 and x_2 subject to $2x_1=0, 3x_2=0$ is isomorphic to $\mathbb{Z}/(6)$.

(03)

Section-C

Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below.

(3 x 6 = 18)

- (i) Show that every subgroup of a solvable group is solvable.
- (ii) State and prove Schur's lemma.
- (iii) Prove that two nilpotent linear transformations $S, T \in A(V)$ are similar if and only if they have same invariants.
- (iv) Show that if U and V be two vector spaces over a field F of dimensions m and n respectively, then $\text{HOM}(U, V)$ is vector space over F of dimension mn .

- (v) Prove that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ is nilpotent, and find the invariants and Jordan form.

Section-D

Long answer type questions (word limit – 500.600 words) Attempt any three questions out of five given below.

(3 x 11 = 33)

- (i) State and Jordan-Holder theorem for finite group.
- (ii) Prove that M be a finitely generated free module over a commutative ring R , then all bases of M have the number of elements.
- (iii) State and prove "Fundamental theorem of R -Homomorphism.

- (iv) Let V be an n -dimensional vector space over a field F and let $T \in A(V)$ has all its characteristics roots leigen value in F . then there is a basis of V in which the matrix of T is triangular.

(06)

- (v) Find the invariant factors of the matrix $\begin{bmatrix} -x & 4 & -2 \\ -3 & 8-x & 3 \\ 4 & -8 & -2-x \end{bmatrix}$

Over the ring $Q[x]$. Also find the rank.

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PGS-251023
Autonomous Examination-2025
SEMESTER-I
M.Sc. - MATHEMATICS
Paper – III (Topology-I)

Time : 3 Hours

M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

Objective type questions Attempt any 10 questions out of 12 given below.

(1 x 10 = 10)

- (i) The set $N \times N$ is :
 (a) Finite (b) Countable
 (c) Uncountable (d) None of these
- (ii) The union of two topologies is :
 (a) Necessarily topology (b) Not necessary topology
 (c) May or may not be topology (d) None of these
- (iii) If each point X possesses a countable local base then such space is said to be :
 (a) Dense (b) First countable
 (c) Second countable (d) None of these
- (iv) The closure of connected set is :
 (a) Totally disconnected (b) Partially connected
 (c) Connected (d) Disconnected
- (v) Every compact subset A to Hausdorff space X is :
 (a) Open (b) Close
 (c) Has limit point (d) None of these
- (vi) A property of a topological space is said to be if every sub-space of the space has that property.
- (vii) Every uniform continuous map is
- (viii) Everysubspace of Lindeloff space is lindeloff space.
- (ix) "Every non-void order set in which each chain has an upper bound has a maximal elements" is the statement of

(02)

- (x) The set $\{x \in X : \text{each nbd of } x \text{ intersect } A \text{ where } A \subset X\}$ is known as.....
- (xi) The limit point of $]0, 1[$ is $[0, 1]$. (True/False)
- (xii) In separable spaces : $T_5 \Rightarrow T_4 \Rightarrow T_3 \Rightarrow T_2 \Rightarrow T_1 \Rightarrow T_0$ (True/False)

Section-B

Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below.

(3 x 3 = 09)

- (i) Prove that the set of all real numbers in the open interval $(0, 1)$ is not enumerable.
- (ii) Define neighbourhood and neighbourhood system of a point in a topological space.
- (iii) Show that a subspace of a topological space is itself a topological space.
- (iv) Define countable compact space and sequentially compact space.
- (v) Prove that every metric space is a T_2 -space.

Section-C

Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below. Question for annotation is compulsory.

(6 x 3 = 18)

- (i) Let A be subset of topological space, then prove that :
 $\bar{A} = A \cup D(A)$
 Where $D(A)$ is derived set.
- (ii) Prove that every compact subset of Hausdorff space is closed.
- (iii) Prove that a map $f : (X, d) \rightarrow (Y, p)$ is continuous if and only if
 $f(\bar{A}) \subset \overline{f(A)} \quad \forall A \subset X$.
- (iv) State & prove Urysohn's lemma.
- (v) Prove that every T_2 – space is T_1 – space but converse need not be true.

(03)

Section-D

Long answer type questions (word limit – 500-600 words) Attempt any three questions out of five given below.

(11 x

03 = 33)

- (i) State & prove Schroeder- Bernstein theorem.
- (ii) Show that every second countable Space is separable.
- (iii) State and prove Lindeloff theorem.
- (iv) State and prove Tietze extension theorem.
- (v) If (X, d) be a , metric space and $\mathfrak{T}$ consists of Φ and all those subset G of X having property that to each $x \in A \exists r > 0$ such that $S(x, r) \subset A$, then $\mathfrak{T}$ is a topology of X .

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PGS-251072
Autonomous Examination DEC-2025
SEMESTER-I
M.A./M.Sc./M.Com./LLM
Paper – IV - (Research Methodology & Computer
Application:Basics)

Time : 3 Hours

M.M. : 70

नोट : सभी प्रश्न हल करना अनिवार्य है। प्रश्न के समक्ष अंक एवं शब्द सीमा निर्दिष्ट है।

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

‘खण्ड-अ’

Section-A

1.वस्तुनिष्ठ प्रश्न-दिये गये 12 प्रश्नों में से किन्हीं 10 प्रश्नों के उत्तर दीजिए।

Objective type questions Attempt any 10 questions out of 12 given below.

(01 x 10 = 10)

- (i) निम्न में से कौन सा शोध का अंग नहीं है-
- (अ) परिकल्पना निर्माण (ब) तथ्यों का संकलन
 (स) परिकल्पना का परीक्षण (द) साहित्यिक चोरी
- Which of the following is not a part of research?
- (a) Hypothesis generation (b) Data collection
 (c) Hypothesis testing (d) Plagiarism
- (ii) शोध-नैतिकता में शामिल है-
- (अ) ईमानदारी और सत्यनिष्ठा (ब) पारदर्शिता
 (स) निष्पक्षता (द) उपरोक्त सभी
- Research ethics includes -
- (a) Honesty and Integrity (b) Transparency
 (c) Impartiality (d) All of the above
- (iii) किसी विशिष्ट उद्देश्य के लिए शोधकर्ता द्वारा एकत्र की गई मूल जानकारी कहलाती है-
- (अ) प्राथमिक स्रोत (ब) द्वितीयक स्रोत
 (स) दोनों अ और ब (द) इनमें से कोई नहीं
- Original information collected directly by the researcher for a unique purpose-
- (a) Primary data (b) Secondary data
 (c) Both 'a' & 'b' (d) None of these
- (iv) कंप्यूटर का बाहरी ढांचा कहलाता है-
- (अ) सॉफ्टवेयर (ब) हार्डवेयर
 (स) प्रिंटर (द) इनमें से कोई नहीं

(02)

The outer structure of a computer is called-

- (a) Software (b) Hardware
 (c) Printer (d) None of these

(v) प्रतिचयन का प्रकार है-

- (अ) संभाव्यता प्रतिचयन (ब) गैर-संभाव्यता प्रतिचयन
 (स) दोनों (द) इनमें से कोई नहीं

The type of sampling is-

- (a) Probability sampling (b) Non-probability sampling
 (c) Both (d) None of these

(vi) निम्न में से कौन सा इनपुट उपकरण नहीं है-

- (a) माउस (b) प्रिंटर
 (c) कीबोर्ड (d) स्कैनर

Which of the following is not an input device

- (a) Mouse (b) Printer
 (c) Keyboard (d) Scanner

(vii) एम. एस.पावर प्वाइंट एकसॉफ्टवेयर है।

M.S. Power point is asoftware

(viii) शोध प्रश्न को हल करने के लिए डेटा इकट्ठा करने और उसका

विश्लेषण करने के तरीके कोकहते हैं।

The method of collecting and analyzing data to answer a research question is called

(ix) किसी बड़े समूह से शोध हेतु चुने गये प्रतिनिधिकहलाते हैं।

Representatives selected from a large group for research are called.....

(x) शोध उद्देश्य की पूर्ति हेतु शोध में नैतिकता अत्यंत आवश्यक है। (सत्य/असत्य)

Research Ethics is essential to achieve the research objective.

(True/False)

(xi) साहित्य-समीक्षा शोध का अंग नहीं है।

(सत्य/असत्य)

Literature review is not a part of research.

(True/False)

(xii) गूगल,बिंग,याहू आदि प्रमुख सर्च इंजन हैं।

(सत्य/असत्य)

Google, Bing, Yahoo etc. are the major search engines. (True/False)

(03)

‘खण्ड-ब’

Section-B

2. अति लघुत्तरीय प्रश्न (शब्द सीमा 70–100 शब्द) दिये गये पाँच प्रश्नों में से किन्हीं तीन प्रश्नों के उत्तर दीजिए।

Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below. (3 x 3 = 09)

- (i) शोध के उद्देश्य को स्पष्ट कीजिये |
Explain the purpose of the research.
- (ii) साहित्यिक चोरी से क्या अभिप्राय है?
What is meant by plagiarism?
- (iii) सैम्पल साइज किसे कहते हैं ?
What is sample size?
- (iv) ऑपरेटिंग सिस्टम क्या है ?
What is an operating system?
- (v) शोध-अन्तराल क्या है?
What is Research-Gap?

‘खण्ड-स’

Section-C

3. लघु उत्तरीय प्रश्न (शब्द सीमा 200–250 शब्द) दिये गये पाँच प्रश्नों में से किन्हीं तीन प्रश्नों के उत्तर दीजिए।

Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below. (6 x 3 = 18)

- (i) शोध की परिभाषा लिखिए एवं शोध के विभिन्न चरणों पर प्रकाश डालिए |
Write the definition of research and highlight the various stages of research..
- (ii) साहित्य-समीक्षा से क्या अभिप्राय है? शोध हेतु साहित्य-समीक्षा के महत्व पर प्रकाश डालिए |
What is meant by a literature review? Explain the importance of a literature review for research
- (iii) कंप्यूटर की संरचना का परिचय प्रस्तुत कीजिये।
Give an introduction to the structure of a computer.
- (iv) मात्रात्मक एवं गुणात्मक शोध को उदाहरण सहित परिभाषित कीजिये।
Define quantitative and qualitative research with examples..
- (v) शोध-पत्र प्रकाशन हेतु किन बातों का ध्यान रखा जाना आवश्यक है।
What are the things that need to be kept in mind for publishing a research paper?

(04)

‘खण्ड-द’

Section-D

4. दीर्घोत्तरीय प्रश्न (शब्द सीमा 500–600 शब्द) दिये गये पाँच प्रश्नों में से किन्हीं तीन प्रश्नों के उत्तर दीजिए।

Long answer type questions (word limit – 500-600 words) Attempt any three questions out of five given below. (11 x 3 = 33)

- (i) शोध की विभिन्न पद्धतियों पर प्रकाश डालिए।
Highlight various methods of research.
- (ii) शोध-नैतिकता क्या है? शोध-नैतिकता के वैशिष्ट्य पर प्रकाश डालिए।
What is research ethics? Highlight the salient features of research ethics.
- (iii) शोध-प्रारूप किसे कहते हैं? शोध प्रारूप के विभिन्न अंगों पर प्रकाश डालिए।
What is a research design? Explain the various components of a research design.
- (iv) ‘शोध कार्य में कंप्यूटर की भूमिका’ विषय पर निबंध लिखिए |
Write an essay on the topic ‘Role of computer in research work’.
- (v) शोध के विभिन्न उपकरणों और उनके महत्व पर विस्तृत टिप्पणी कीजिये।
Comment in detail on various tools of research and their importance.

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PGS-251024
Autonomous Examination DEC-2025
SEMESTER-I
M.Sc. - Mathematics
Paper – V (Complex Analysis-I)

Time : 3 Hours

M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

01.Objective type questions Attempt any 10 questions out of 12 given below.
(10 x 01 = 10)

- (i) Simple curve are also called.
 - (a) Multiple curve
 - (b) Jordan curve
 - (c) Integral curve
 - (d) None
- (ii) The pole of $f(z) = \frac{z^2+1}{1-z^2}$ is-
 - (a) 1
 - (b) -1
 - (c) ± 1
 - (d) 0
- (iii) If $f(a) = 0$ and $f'(a) \neq 0$ then $z = a$ is called a -
 - (a) Simple zero
 - (b) Simple curve
 - (c) Zero of order
 - (d) None
- (iv) The residue of $f(z) = \cot z$ at each pair is -
 - (a) 0
 - (b) 1
 - (c) $\frac{1}{2}$
 - (d) None
- (v) Find the invariant points of the bilinear transformation $w = \frac{z-1}{z+1}$
 - (a) 1, -1
 - (b) $i, -i$
 - (c) 0, 1
 - (d) 0, ∞
- (vi) A region which is not simply connected is called region.
- (vii) A point Z_0 at which a function $f(z)$ is not analytic is known as a of $f(z)$.
- (viii) A singular point $z=z_0$ is said to be ansingular point of $f(z)$. If there is no other singular point in the neighborhood of z_0 .
- (ix) The derivative is zero or undefined at the
- (x) The inverse of a bilinear transformation is also a.....

(02)

- (xi) The pole of $f(z) = \frac{z-2}{z^2} \sin\left(\frac{1}{z-1}\right)$ is 2. [True/False]
- (xii) A zero of an analytic function $f(z)$ is a value of z for which $f(z) = 0$. [True/False]

Section-B

02.Very Short answer type questions (word limit – 70-100 words) Attempt any Three questions out of Five given below.
(3 x 3 = 09)

- (i) Evaluate $\int_c \frac{z^2 - z + 1}{z - 1} dz$ where c is the circle $|Z| = 1$.
- (ii) Find the zero and discuss the nature of singularity of the function.
 $f(z) = \frac{z-2}{z^2} \sin\left(\frac{1}{z-1}\right)$
- (iii) Write Schwarz lemma.
- (iv) Find the residue of $\frac{z^2}{z^2 + a^2}$ at $z = ia$.
- (v) Define cross ratio.

Section-C

03.Short answer type questions (word limit – 200-250 words) Attempt any Three questions out of Five given below. Question for annotation is compulsory.
(3 x 6 = 18)

- (i) For $Z \neq 0, 1, \dots, -1$ prove that.

$$\overline{\gamma(Z)} = \lim_{n \rightarrow \infty} \frac{|n \quad n^2}{z(z+1) \dots (z+n)} T(z)$$
- (ii) Prove that cross ratios are invariant under a bilinear transformation.
- (iii) Prove that the set of all bilinear transformation forms a non-abelian group under the product of transformations.
- (iv) Evaluate : $\int_0^\infty \frac{\cos^2 x}{(1+x^2)^2} dx$
- (v) Write and prove Cauchy Goursat theorem.

(03)

Section-D

04. Long answer type questions (word limit – 500–600 words) Attempt any three questions out of five given below.

(3 x 11 = 33)

- (i) State and prove Riemann mapping theorem.
- (ii) State and prove Runge's theorem
- (iii) State and prove Maximum Modulus principle.
- (iv) State and prove Morera's theorem.
- (v) State and prove Taylor's theorem.

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PGS-251022
Autonomous Examination DEC-2025
SEMESTER-I
M. Sc. MATHEMATICS
Paper –II (REAL ANALYSIS – I)

Time : 3 Hours

M.M. : 70

Note: Attempt all questions. Scheme of questions and marks are indicated against the questions.

Section-A

01. Objective type questions Attempt any 10 questions out of 12 given below.
(1 x 10 = 10)

- (i) The Riemann–Stieltjes integral $\int_a^b f dg$ exists if
 - (a) f is continuous and g is of bounded variation
 - (b) g is continuous and f is of bounded variation
 - (c) Both (a) and (b)
 - (d) Neither
- (ii) A sequence of functions $\{f_n\}$ converges uniformly to f on a set E if
 - (a) $\sup |f_n(x)| \rightarrow 0$
 - (b) $\sup |f_n(x) - f(x)| \rightarrow 0$
 - (c) $|f_n(x) - f(x)| \rightarrow 0$ pointwise
 - (d) f_n are monotone
- (iii) The radius of convergence R of $\sum_{n=0}^{\infty} n! x^n$ is
 - (a) 0
 - (b) 1
 - (c) ∞
 - (d) 2
- (iv) If a function $f(x, y)$ is differentiable at (a, b) then
 - (a) all partial derivatives exist there
 - (b) it must be continuous at (a, b)
 - (c) Jacobian must be 0
 - (d) both (a) and (b)
- (v) Stokes' theorem relates
 - (a) surface integral to line integral
 - (b) double integral to single integral
 - (c) triple integral to surface integral
 - (d) none
- (vi) The Fundamental Theorem of Calculus connects the integral with the _____ of a function.
- (vii) A necessary and sufficient condition for uniform convergence of series $\sum f_n$ is given by _____ test.
- (viii) The necessary condition for a function $f(x, y)$ to have an extremum at a point is
- (ix) The Implicit Function Theorem requires the Jacobian determinant to be _____ at the point.
- (x) Lagrange's multiplier method is used to find extrema subject to _____.

(02)

- (xi) Every pointwise convergent sequence of continuous functions is uniformly convergent. (True/False)
- (xii) If partial derivatives exist at a point, the function must be differentiable there. (True/False)

Section-B

02. Short answer type questions (word limit -100 words) Attempt any Three questions out of Five given below. (3 x 03= 09)

- (i) Define Riemann - Stieltjes integral with an example.
- (ii) State Weierstrass M-test and use it to check whether $\sum \frac{\sin(nx)}{n^2}$ converges uniformly or not.
- (iii) Explain power series and its radius of convergence with an example.
- (iv) State the Chain Rule for functions of several variables and illustrate with a simple numerical example.
- (v) If $u = x - y, v = x + y$ then determine $\frac{\partial(u,v)}{\partial(x,y)}$.

Section-C

03. Middle answer type questions (word limit -250 words) Attempt any Three questions out of Five given below. (6 x 03= 18)

- (i) Show that a function f is RS integrable with respect to α on $[a, b]$ if and only if for every $\epsilon > 0$, there exists a partition P of $[a, b]$, such that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.
- (ii) State and prove Cauchy's criterion for uniform convergence of a sequence of functions.
- (iii) Let $\sum_{n=1}^{\infty} U_n(x)$ be a uniformly convergent series of continuous functions on $[a, b]$. Show that

$$\int_a^b \left(\sum_{n=1}^{\infty} U_n(x) \right) dx = \sum_{n=1}^{\infty} \int_a^b U_n(x) dx.$$

- (iv) Show that the function $f(x, y) = \sqrt{|xy|}$ is not differentiable at origin, although both the first partial derivatives f_x and f_y exist there. Are f_x and f_y continuous at origin?

(03)

- (v) The roots of the equation in λ ,
 $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$,
 are u, v and w . Prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2 \frac{(y - z)(z - x)(x - y)}{(v - w)(w - u)(u - v)}.$$

Section-D

04. Long answer type questions (word limit -750 words) Attempt any three questions out of five given below. (11 x 03 = 33)

- (i) If f is continuous and α is monotone on $[a, b]$ then prove that

$$\int_a^b f d\alpha = [f(x)\alpha(x)]_a^b - \int_a^b \alpha df.$$

- (ii) Prove that a series obtained from an absolutely convergent series by a rearrangement of terms converges absolutely and has the same sum as the original series.

- (iii) State and prove Abel's limit theorem for power series and hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$$

- (iv) State Taylor's theorem for functions of two variables and hence expand $f(x, y) = e^x \cos y$ near the point $(1, \frac{\pi}{4})$.

- (v) Use the method of Lagrange's multipliers to find the volume of largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

